## Final Exam, MTH 211, Spring 2010

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**QUESTION 1.** Let  $L_1$  and  $L_2$  be two lines intersect in an angle  $\alpha$  such that  $\alpha \neq 90$ . USED UNMARKED RULER AND A COMPASS TO BISECT THE ANGLE  $\alpha$ . STATE CLEARLY THE STEPS OF CONSTRUCTIONS (NO MATH JUSTIFICATION IS NEEDED)

**QUESTION 2.** Let *abcd* be a square with *ad* as the base and *cd* as the width. Let *m* be the midpoint of *ad*. Draw a circle *C* centered at *m* with radius *cm*. Then *C* intersects the extended line of *ad* at a point *k*. Prove that ak/ad = the Golden Ratio.

**QUESTION 3.** USE UNMARKED RULER AND A COMPASS TO CONSTRUCT A GOLDEN CUTE TRIAN-GLE with base that has length 4*cm* (you may use a marked ruler to measure 4cm).

**QUESTION 4.** Let  $L_1$  and  $L_2$  be two perpendicular lines. Choose a point m such that m does not lie on  $L_1$  and m does not lie on  $L_2$ . USE UNMARKED RULER AND A COMPASS to find a point, say a, on the line  $L_1$  and a point, say b on  $L_2$  so that the line segment ab passes through m and |am| = 2 |mb|. STATE THE STEPS OF CONSTRUCTIONS. THEN VERIFY YOUR CONSTRUCTION.

**QUESTION 5.** Let C be a circle of radius 3 and center O. Let A be a point inside C such that |OA| = 1cm. a) Is there a circle D of radius 3.5cm such that D passes through A and orthogonal to C? if yes, do nothing. If no, then explain why not.

b) USE a marked ruler and a compass to construct a circle F of radius  $\sqrt{17}$  such that D passes through A and orthogonal to C.

**QUESTION 6.** Let *C* be a circle of radius 2 and center *O*. Let *A* be a point such that |OA| = 1. Let *D* be a circle orthogonal to *C* and centered at Inv(A). Let *m* be the intersection point of *D* with the line segment OInv(A). Find the exact length of the line segment Inv(A)Inv(D).

QUESTION 7. (i) Can we construct an angle of 10 degrees (using unmarked ruler and a compass)? EXPLAIN

(ii) Can we construct a regular 22-gon (using unmarked ruler and compass)? explain

(iii) I claim that we can construct a regular 40-gon. Justify my claim. What will be the measurement of each interior angle?

(iv) Three types of regular gon: Say K, M, N. The K-type is regular 12 gon. We must use at least one piece of each type in order to tile a plane. What are the possibilities for the M-type and the N-type? State all possibilities WITHOUT ANY JUSTIFICATION. **QUESTION 8.** (i) Let H be a hyperbolic circle with radius 3cm and center O. Let A be a point inside H such that  $d_h(O, A) = \ln(5)$  (the hyperbolic distance is  $\ln(5)$ ). Find d(O, A) (the Euclidean distance between O and A). Show the work

(ii) Let H be a hyperbolic circle with radius 3cm and center O. Let B be a horizon point on H. Now choose two points A, C inside H such that d(O, A) = d(O, C) = 1 cm. Given A, B do not lie on a diameter of H, and A, C do not lie on a diameter of H, and C, B do not lie on a diameter of H. Construct two lines say,  $L_1, L_2$ , such that  $L_1$  passes through A,  $L_2$  passes through C,  $L_1$  is parallel to  $L_2$  but  $L_1$  meets  $L_2$  at B. STATE THE STEPS OF CONSTRUCTIONS WITHOUT ANY MATH JUSTIFICATION.

## **Faculty information**

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