# Final Exam , MTH 211, Spring 2010 

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QUESTION 1. Let $L_{1}$ and $L_{2}$ be two lines intersect in an angle $\alpha$ such that $\alpha \neq 90$. USED UNMARKED RULER AND A COMPASS TO BISECT THE ANGLE $\alpha$. STATE CLEARLY THE STEPS OF CONSTRUCTIONS (NO MATH JUSTIFICATION IS NEEDED)

QUESTION 2. Let $a b c d$ be a square with $a d$ as the base and $c d$ as the width. Let $m$ be the midpoint of $a d$. Draw a circle $C$ centered at $m$ with radius $c m$. Then $C$ intersects the extended line of $a d$ at a point $k$. Prove that $a k / a d=$ the Golden Ratio.

QUESTION 3. USE UNMARKED RULER AND A COMPASS TO CONSTRUCT A GOLDEN CUTE TRIANGLE with base that has length 4 cm (you may use a marked ruler to measure 4 cm ).

QUESTION 4. Let $L_{1}$ and $L_{2}$ be two perpendicular lines. Choose a point $m$ such that $m$ does not lie on $L_{1}$ and $m$ does not lie on $L_{2}$. USE UNMARKED RULER AND A COMPASS to find a point, say $a$, on the line $L_{1}$ and a point, say $b$ on $L_{2}$ so that the line segment $a b$ passes through $m$ and $|a m|=2|m b|$. STATE THE STEPS OF CONSTRUCTIONS. THEN VERIFY YOUR CONSTRUCTION.

QUESTION 5. Let $C$ be a circle of radius 3 and center $O$. Let $A$ be a point inside $C$ such that $|O A|=1 \mathrm{~cm}$.
a) Is there a circle $D$ of radius 3.5 cm such that $D$ passes through $A$ and orthogonal to $C$ ? if yes, do nothing. If no, then explain why not.
b) USE a marked ruler and a compass to construct a circle $F$ of radius $\sqrt{17}$ such that $D$ passes through $A$ and orthogonal to $C$.

QUESTION 6. Let $C$ be a circle of radius 2 and center $O$. Let $A$ be a point such that $|O A|=1$. Let $D$ be a circle orthogonal to $C$ and centered at $\operatorname{Inv}(A)$. Let $m$ be the intersection point of $D$ with the line segment $\operatorname{OInv}(A)$. Find the exact length of the line segment $\operatorname{Inv}(A) \operatorname{Inv}(D)$.

QUESTION 7. (i) Can we construct an angle of 10 degrees (using unmarked ruler and a compass)? EXPLAIN
(ii) Can we construct a regular 22-gon (using unmarked ruler and compass)? explain
(iii) I claim that we can construct a regular 40-gon. Justify my claim. What will be the measurement of each interior angle?
(iv) Three types of regular gon: Say K, M, N. The K-type is regular 12 gon. We must use at least one piece of each type in order to tile a plane. What are the possibilities for the M-type and the N-type? State all possibilities WITHOUT ANY JUSTIFICATION.

QUESTION 8. (i) Let $H$ be a hyperbolic circle with radius 3 cm and center O . Let $A$ be a point inside $H$ such that $d_{h}(O, A)=\ln (5)$ (the hyperbolic distance is $\ln (5)$ ). Find $d(O, A)$ (the Euclidean distance between O and A). Show the work
(ii) Let $H$ be a hyperbolic circle with radius 3 cm and center O . Let $B$ be a horizon point on $H$. Now choose two points $A, C$ inside $H$ such that $d(O, A)=d(O, C)=1 \mathrm{~cm}$. Given $A, B$ do not lie on a diameter of $H$, and $A, C$ do not lie on a diameter of $H$, and $C, B$ do not lie on a diameter of $H$. Construct two lines say, $L_{1}, L_{2}$, such that $L_{1}$ passes through A, $L_{2}$ passes through $C, L_{1}$ is parallel to $L_{2}$ but $L_{1}$ meets $L_{2}$ at $B$. STATE THE STEPS OF CONSTRUCTIONS WITHOUT ANY MATH JUSTIFICATION.

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